

# 1-2

## Properties of Real Numbers

### Common Core State Standards

**Reviews N-RN.B.3** Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational, and that the product of a nonzero rational number and an irrational number is irrational.

MP 1, MP 3, MP 6

**Objectives** To graph and order real numbers  
To identify properties of real numbers



**SOLVE IT!** Getting Ready!

You use emoticons in text messages to help you communicate. Here are six emoticons. How can you describe a set that includes five of the emoticons but not the sixth?

**Lesson Vocabulary**

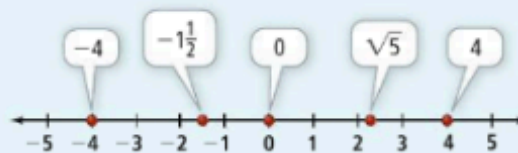
- opposite
- additive inverse
- reciprocal
- multiplicative inverse

In the Solve It, you classified sets of emoticons. In this lesson, you will classify real numbers into special subsets.

**Essential Understanding** The set of real numbers has several special subsets related in particular ways.

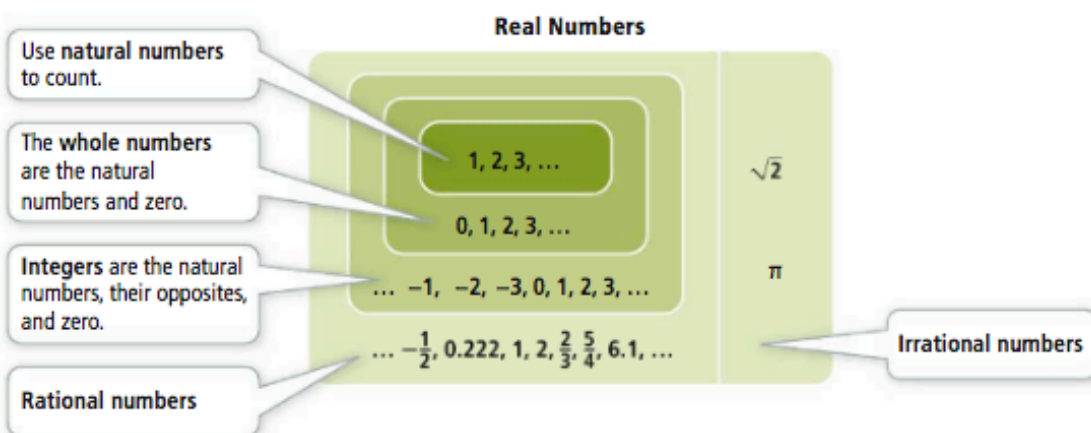
Algebra involves operations on and relations among numbers, including real numbers and imaginary numbers. (You will learn about imaginary numbers in Chapter 4.) Rational numbers and irrational numbers form the set of real numbers.

You can graph every real number as a point on the number line.





The diagram shows how subsets of the real numbers are related.



#### Rational numbers

- are all numbers you can write as a quotient of integers  $\frac{a}{b}$ ,  $b \neq 0$ .
- include terminating decimals. For example,  $\frac{1}{8} = 0.125$ .
- include repeating decimals. For example,  $\frac{1}{3} = 0.\overline{3}$ .

#### Irrational numbers

- have decimal representations that neither terminate nor repeat. For example,  $\sqrt{2} = 1.414213 \dots$
- cannot be written as quotients of integers.

You classify a variable by naming the subset that gives you the most information about the numbers the variable represents.



### Problem 1 Classifying a Variable

**Multiple Choice** Your school is sponsoring a charity race. Which set of numbers does not contain the number of people  $p$  who participate in the race?

- (A) natural numbers      (C) rational numbers  
(B) integers                (D) irrational numbers

The number of people  $p$  is a natural number, which means that it is also an integer and a rational number. The correct answer is D.



**Got It?** 1. In Problem 1, if each participant made a donation  $d$  of \$15.50 to a local charity, which subset of real numbers best describes the amount of money raised?

### Think

What are some examples of possible values of  $p$ ?

The number of people  $p$  must be represented by a whole number. Determine which other sets of numbers include the whole numbers.



### Problem 2 Graphing Numbers on the Number Line

What is the graph of the numbers  $-\frac{5}{2}$ ,  $\sqrt{2}$ , and  $2.\bar{6}$ ?

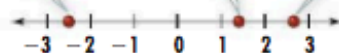
#### Plan

How do you graph a number on the number line? If the number is an integer, determine whether it is positive or negative. If it's not an integer, determine which integer it's closest to.

Since  $-\frac{5}{2} = -2\frac{1}{2}$ ,  $-\frac{5}{2}$  is between  $-3$  and  $-2$ .

Use a calculator.  $\sqrt{2} \approx 1.4$ .

Think:  $2.\bar{6} = 2\frac{2}{3}$ .



**Got It?** 2. What is the graph of the numbers  $\sqrt{3}$ ,  $-1.\bar{4}$ , and  $\frac{1}{3}$ ?

The number line is helpful for ordering several real numbers. For two numbers, however, it is easier to show order, or compare, using one of the inequality symbols  $>$  or  $<$ .



### Problem 3 Ordering Real Numbers

How do  $\sqrt{17}$  and  $3.8$  compare? Use  $>$  or  $<$ .

#### Think

Why compare  $\sqrt{17}$  to the square root of a perfect square? It makes it easier to determine which two integers  $\sqrt{17}$  is between.

Compare both numbers to  $\sqrt{16}$ .

$$\sqrt{16} < \sqrt{17} \quad 16 \text{ is less than } 17.$$

$$3.8 < \sqrt{16} \quad \sqrt{16} = 4 \text{ and } 3.8 < 4.$$

Therefore,  $3.8 < \sqrt{17}$ , or  $\sqrt{17} > 3.8$ .

**Check** Use a calculator.

$$\sqrt{17} \approx 4.123$$

$$3.8 < 4.123 \quad \checkmark$$



**Got It?** 3. a. How do  $\sqrt{26}$  and  $6.25$  compare? Use  $>$  or  $<$ .

b. **Reasoning** Let  $a$ ,  $b$ , and  $c$  be real numbers such that  $a < b$  and  $b < c$ . How do  $a$  and  $c$  compare? Explain.

**Essential Understanding** The properties of real numbers are relationships that are true for all real numbers (except, in one case, zero).

One property of real numbers excludes a single number, zero. Zero is the *additive identity* for the real numbers, and zero is the one real number that has no *multiplicative inverse*.





The **opposite** or **additive inverse** of any number  $a$  is  $-a$ .  
The sum of a number and its opposite is 0, the **additive identity**.

**Examples**  $12 + (-12) = 0$        $-7 + 7 = 0$

The **reciprocal** or **multiplicative inverse** of any nonzero number  $a$  is  $\frac{1}{a}$ .  
The product of a number and its reciprocal is 1, the **multiplicative identity**.

**Examples**  $8\left(\frac{1}{8}\right) = 1$        $-5\left(-\frac{1}{5}\right) = 1$



### Properties Properties of Real Numbers

Let  $a$ ,  $b$ , and  $c$  represent real numbers.

#### Property

#### Addition

#### Multiplication

Closure

$a + b$  is a real number.

$ab$  is a real number.

Commutative

$a + b = b + a$

$ab = ba$

Associative

$(a + b) + c = a + (b + c)$

$(ab)c = a(bc)$

Identity

$a + 0 = a$ ,  $0 + a = a$   
0 is the additive identity.

$a \cdot 1 = a$ ,  $1 \cdot a = a$   
1 is the multiplicative identity.

Inverse

$a + (-a) = 0$

$a \cdot \frac{1}{a} = 1$ ,  $a \neq 0$

Distributive

$a(b + c) = ab + ac$



### Problem 4 Identifying Properties of Real Numbers

Which property does the equation illustrate?

**A**  $\left(-\frac{2}{3}\right)\left(-\frac{3}{2}\right) = 1$

The product of the numbers is 1.  
Inverse Property of Multiplication

**B**  $(3 \cdot 4) \cdot 5 = (4 \cdot 3) \cdot 5$

The equation reorders 3 and 4.  
Commutative Property of Multiplication



**Got It?** 4. a. Which property does the equation  $3(g + h) + 2g = (3g + 3h) + 2g$  illustrate?

b. **Reasoning** Use properties of real numbers to show that  $a + [3 + (-a)] = 3$ . Justify each step of your solution.

### Plan

How can you analyze an equation?

Determine whether it

- uses addition or multiplication
- reorders or regroups the numbers
- uses an identity



## Lesson Check

### Do you know HOW?

Write an example from daily life that uses each type of real number.

- whole numbers
- integers
- rational numbers

Identify the property illustrated by the equation.

- $5 + (-5) = 0$
- $2 \cdot (4 \cdot 5) = (2 \cdot 4) \cdot 5$

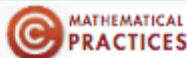
### Do you UNDERSTAND?



- Vocabulary** Identify another name for a reciprocal.
- Compare and Contrast** How is the Additive Identity Property similar to the Multiplicative Identity Property? How is it different?
- Reasoning** There are grouping symbols in the equation  $(5 + w) + 8 = (w + 5) + 8$ , but it does not illustrate the Associative Property of Addition. Explain.
- Give an example of a number that is not a rational number. Explain why it is not rational.



## Practice and Problem-Solving Exercises



### Practice

Classify each variable according to the set of numbers that best describes its values.

See Problem 1.

- the number of times  $n$  a ball bounces; the height  $h$  from which the ball is dropped
- the year  $y$ ; the median selling price  $p$  for a house that year
- the circumference  $C$  of a circle found by using the formula  $C = 2\pi r$

Graph each number on a number line.

See Problem 2.

- |         |                  |                 |                     |                     |
|---------|------------------|-----------------|---------------------|---------------------|
| 13. 0   | 14. $-\sqrt{24}$ | 15. $-2$        | 16. $2\frac{1}{2}$  | 17. $-4\frac{2}{3}$ |
| 18. 3.5 | 19. $-1.4$       | 20. $\sqrt{10}$ | 21. $-2\frac{1}{5}$ | 22. 4.8             |

Compare the two numbers. Use  $>$  or  $<$ .

See Problem 3.

- |                            |                     |                          |
|----------------------------|---------------------|--------------------------|
| 23. 16, $\sqrt{16}$        | 24. $-4, -\sqrt{4}$ | 25. $\sqrt{5}, \sqrt{7}$ |
| 26. $-\sqrt{3}, -\sqrt{5}$ | 27. 5, $\sqrt{22}$  | 28. $-\sqrt{38}, 6$      |
| 29. 4, $\sqrt{12}$         | 30. $-8, \sqrt{70}$ | 31. $\sqrt{63}, 7.5$     |
| 32. 4.7, $\sqrt{26}$       | 33. $\sqrt{75}, 9$  | 34. 12, $-\sqrt{150}$    |

Name the property of real numbers illustrated by each equation.

See Problem 4.

- |   |   |
|---|---|
| 35. $\pi(a + b) = \pi a + \pi b$                              | 36. $-10 + 4 = 4 + (-10)$               |
| 37. $(2\sqrt{7}) \cdot \sqrt{3} = 2(\sqrt{7} \cdot \sqrt{3})$ | 38. $29 \cdot \pi = \pi \cdot 29$       |
| 39. $-\sqrt{5} + 0 = -\sqrt{5}$                               | 40. $\frac{4}{7} \cdot \frac{7}{4} = 1$ |

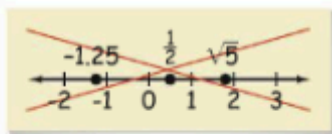
**B** Apply

Estimate the numbers graphed at the labeled points.



41. point A                      42. point B                      43. point C                      44. point D  
45. point E                      46. point F                      47. point G                      48. point H

49. **Think About a Plan** A cube-shaped jewelry box has a surface area of 300 square inches. What are the dimensions of the jewelry box?
- Write an algebraic expression to find the total surface area of a cube. What is the surface area of one side of a cube?
  - How is the side length of a square related to its area?
50. **Error Analysis** A student labeled the points on the number line as shown. Explain the student's error.



- STEM Science** The formula  $I = \sqrt{\frac{W}{R}}$  gives the electric current  $I$  in amperes that flows through an appliance, where  $W$  is the power in watts and  $R$  is the resistance in ohms. Which set of numbers best describes the value of  $I$  for the given values of  $W$  and  $R$ ?

51.  $W = 100, R = 25$                       52.  $W = 100, R = 5$                       53.  $W = 500, R = 100$   
54.  $W = 50, R = 200$                       55.  $W = 250, R = 100$                       56.  $W = 240, R = 100$

Write the numbers in decreasing order.

57.  $1, -3, -\sqrt{2}, 8, \frac{1}{3}$                       58.  $\sqrt{14}, \frac{5}{2}, -\frac{9}{16}, 1, 11$                       59.  $-17, -0.06, -3\sqrt{3}, 5.73, \frac{1}{4}$

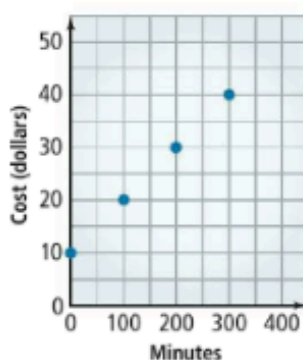
60. **Reasoning** An example is a *counterexample* to a general statement if it makes the statement false. Show that each of the following statements is false by finding a counterexample.
60. The reciprocal of each natural number is a natural number.  
61. The opposite of each whole number is a whole number.  
62. There is no integer that has a reciprocal that is an integer.  
63. The product of two irrational numbers is an irrational number.  
64. All square roots are irrational numbers.
65. **Writing** Write an example of each of the 11 properties of real numbers shown on page 14.
66. **Restaurant** Five friends each ordered a sandwich and a drink at a restaurant. Each sandwich costs the same amount and each drink costs the same amount. What are two ways to compute the bill? What property of real numbers is illustrated by the two methods?
67. **Open-Ended** Write an algebraic problem that requires the use of the real-number properties to solve. Then solve the problem.

**Challenge**

- 68. Writing** Are there two integers with a product of  $-12$  and a sum of  $-3$ ? Explain.
- 69.** Your friend used the Distributive Property and got the expression  $5x + 10y - 35$ . What algebraic expression could your friend have started with?
- 70. Geometry**  $\pi$  is an irrational number you can use to calculate the circumference or area of a circle.
- Find the value of  $\pi$  on your calculator. Can you obtain an exact representation? Explain.
  - The value of  $\pi$  is often represented as  $\frac{22}{7}$ . How does this representation compare to the decimal representation your calculator gives using the  $\pi$  key?
- 71.** Does zero have a multiplicative inverse? Explain.

**Standardized Test Prep****SAT/ACT**

- 72.** Which of the following shows the numbers  $\pi$ ,  $\sqrt{8}$ , and  $3.5$  in the correct order from greatest to least?
- (A)  $\pi$ ,  $\sqrt{8}$ ,  $3.5$       (B)  $3.5$ ,  $\pi$ ,  $\sqrt{8}$       (C)  $\sqrt{8}$ ,  $\pi$ ,  $3.5$       (D)  $\sqrt{8}$ ,  $3.5$ ,  $\pi$
- 73.** Which of the following is the best statement about the graph?
- (F) A 400-minute plan costs \$40.  
 (G) A 100-minute plan costs \$10.  
 (H) A 1000-minute plan costs \$110.  
 (I) A 200-minute plan costs \$35.

**Cell Phone Plan****Short Response**

- 74.** Why is the opposite of the reciprocal of 5 the same as the reciprocal of the opposite of 5?

**Apply What You've Learned****MATHEMATICAL PRACTICES**  
MP 1

Look back at the information on page 3 about Cody's car running out of gas.

- Copy the diagram shown on page 3. Place a point  $C$  along the line at a point where Cody's car could be when it runs out of gas. Draw as many diagrams as needed to show the possible locations of Cody's car relative to the locations of Mia's house, the gas station, and the restaurant.
- How many diagrams did you draw in part (a)? Explain why you needed that number of diagrams to account for all possible locations of Cody's car relative to the other three locations.